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BOOK OF ABSTRACTS

May to December 2024

SISSA, International School of Advanced Studies
Trieste, Italy

Fractional Calculus Seminar Series

Editors:
Pavan Pranjivan Mehta
Arran Fernandez

SISSA Liaison:
Gianluigi Rozza

Website: https://mathlab.sissa.it/fractional-calculus-seminars
YouTube: https://www.youtube.com/@FractionalCalculusSeminar

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Book of Abstracts
Fractional Calculus Seminar Series

May to December 2024

SISSA, International School of Advanced Studies
Trieste, Italy

Editors

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Preface

Fractional calculus is the field of research that studies fractional derivatives and fractional integrals, which are variously defined as derivatives and integrals to non-integer orders. This is a generalised form of integer-order calculus, with more richness and variety since fractional derivatives and integrals can be defined in many different ways which are not equivalent to each other: there is no single unique answer to a question like "what is the derivative to order one-half of the identity function?"

It is well known that integer-order derivatives are local operators and integer-order integrals are non-local operators. In fractional calculus, since the derivative operators are defined using the integral operators, both fractional integrals and fractional derivatives are non-local operators. Depending on the type of fractional derivative used, these operators may depend on values of the function in a finite region, or in a one-way region modelling a memory effect, or in its entire domain. The non-locality property is one of the reasons why fractional calculus has found many applications: real-world applications of non-local models can be found in turbulence, viscoelasticity, fracture mechanics, economic models, diffusion processes, electrical circuits, and plasma physics. The full range of applications is not yet understood, and new research is ongoing in many of these domains.

The theory and applications of fractional integro-differential operators and equations has not received much attention in the wider scientific community, beyond a few specialists developing the field, so that many fundamental questions remain unanswered and the field is ripe for ongoing research in many directions. Currently, however, there is not a single united research community in fractional calculus, but rather many different groups working on it in different ways. Some research is undertaken without awareness of the established fundamentals of the field or of what other research groups are doing.

Thus, the Fractional Calculus seminar series grew out of the necessity to connect different research communities and to touch on as many aspects of fractional calculus as possible. This seminar series is intended to provide deep knowledge on all aspects of fractional calculus, from analytical mathematics to numerical simulations to modelling applications. Some of the presentations are from long-standing experts who have been working in the field for decades, while some reflect new developments in particular research directions. Some of the topics are of broad interest to anyone working in fractional calculus, but there were also some focus sessions (listed below) which allowed us to drill further into particular broad topics of research in fractional calculus.

Special focus sessions:

- **24 May to 21 June 2024 (7 talks)**: fractional/nonlocal modelling of turbulence.
- **05 July to 02 August 2024 (6 talks)**: stochastic processes and probability theory for fractional PDEs.
- **09 August to 06 September 2024 (5 talks)**: general fractional-calculus operators.
- **13 September to 11 October 2024 (5 talks)**: fractional inverse problems.
- **18 October to 06 December 2024 (10 talks)**: numerical analysis, methods, and singular integral computation.

Pavan Pranjivan Mehta
Arran Fernandez

Fractional Calculus Resources

Books and References

Textbooks on Fractional Calculus

- K.S. Miller, B. Ross, *An Introduction to the Fractional Calculus and Fractional Differential Equations*, John Wiley, New York, 1993.
- K.B. Oldham, J. Spanier, *The Fractional Calculus*, Academic Press, New York, 1974.
- S.G. Samko, A.A. Kilbas, O.I. Marichev, *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach, Yverdon, 1993.
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- K. Diethelm, *The Analysis of Fractional Differential Equations: an Application-oriented Exposition using Differential Operators of Caputo Type*, Springer, Heidelberg, 2010.
- D. Baleanu, K. Diethelm, E. Scalas, J.J. Trujillo, *Fractional Calculus: Models and Numerical Methods*, World Scientific, New Jersey, 2012.
- J.A.T. Machado (ed.), *Handbook of Fractional Calculus with Applications*, de Gruyter, Berlin, 2019.

Numerical and Stochastic Approaches

- M.M. Meerschaert, A. Sikorskii, *Stochastic models for fractional calculus*, de Gruyter, Berlin, 2019.
- K. Diethelm, N.J. Ford, A.D. Freed, Y. Luchko, "Algorithms for the fractional calculus: a selection of numerical methods", *Computer Methods in Applied Mechanics and Engineering* 194 (2005), 743–773.
- C. Li, F. Zeng, *Numerical Methods for Fractional Calculus*, CRC Press, Boca Raton, 2015.
- M. Zayernouri, L.-L. Wang, J. Shen, G.E. Karniadakis, *Spectral and Spectral Element Methods for Fractional Ordinary and Partial Differential Equations*, Cambridge University Press, Cambridge, 2024.

- G. Pang, L. Lu, G.E. Karniadakis, “fPINNs: Fractional physics-informed neural networks”, *SIAM Journal on Scientific Computing* 41(4) (2019), A2603–A2626.

- C.P. Li, M. Cai, *Theory and Numerical Approximations of Fractional Integrals and Derivatives*, SIAM, Philadelphia, 2019.

- Yu Y, Liu N, Lu F, Gao T, Jafarzadeh S, Silling S. Nonlocal attention operator: Materializing hidden knowledge towards interpretable physics discovery. arXiv preprint arXiv:2408.07307. 2024 Aug 14.

Further Reading on Related Topics

- R. Hilfer, Y. Luchko, “Desiderata for fractional derivatives and integrals”, *Mathematics* 7(2) (2019), 149.

- R. Gorenflo, A.A. Kilbas, F. Mainardi, S.V. Rogosin, *Mittag-Leffler Functions, Related Topics and Applications* (2nd ed.), Springer, Berlin, 2020.

- V.E. Tarasov, “Fractional vector calculus and fractional Maxwell’s equations”, *Annals of Physics* 323(11) (2008), 2756–2778.

- A. Lischke, G. Pang, M. Gulian, F. Song, C. Glusa, X. Zheng, Z. Mao, W. Cai, M.M. Meerschaert, M. Ainsworth, G.E. Karniadakis, “What is the fractional Laplacian? A comparative review with new results”, *Journal of Computational Physics* 404 (2020), 109009.

- P. Pranjivan Mehta, “Fractional and tempered fractional models for Reynolds-averaged Navier–Stokes equations”, *Journal of Turbulence* 24 (2023), 507–553.

- D.Y. Xue, L. Bai, *Fractional Calculus: High-Precision Algorithms and Numerical Implementations*, Springer Nature, Singapore, 2024.

- O. Marichev, E. Shishkina, “Overview of fractional calculus and its computer implementation in Wolfram Mathematica”, *Fractional Calculus and Applied Analysis* 27(5) (2024), 1995–2062.

- F. Mainardi, *Fractional Calculus and Waves in Linear Viscoelasticity* (2nd ed.), World Scientific, Singapore, 2022.

- J.-L. Gonzales Santander, F. Mainardi, “Some fractional integral and derivative formulas revisited”, *Mathematics* 12(17) (2024), 2786.

Software and Codes

The below list of software / codes provides non-local / fractional / peridynamics capabilities.

- **Peridigm:** https://github.com/peridigm/peridigm
- **PyNucleus:** https://github.com/sandialabs/PyNucleus
  Documentation : https://sandialabs.github.io/PyNucleus/
- **CabanaPD:** https://github.com/ORNL/CabanaPD
- **PDMATLAB2D:** https://github.com/ORNL/PDMATLAB2D/
- **VaYu:** https://mehta-pavan.github.io/vayu/
- **IMEX_FDES:** https://github.com/suzukijo/IMEX_FDEs
- **RHEOS:** https://github.com/JuliaRheology/RHEOS.jl

Other popular open-source software projects

- **Deal.II:** https://www.dealii.org/
- **NekTar++:** https://www.nektar.info/
- **libParanumal:** https://www.paranumal.com/software
- **FEniCS:** https://fenicsproject.org/
- **NekRS:** https://github.com/Nek5000/nekRS

Contents

Abstracts

03 May 2024
14:30-15:30
CEST

The general Fractional Calculus operators with the Sonin kernels: Basic properties, applications, and history of origins

Yuri Luchko

Berlin University of Applied Sciences and Technology, Germany

Time: 14:30 - 15:30 CEST (Rome / Paris)
Date: 03 May 2024
YouTube: https://youtu.be/0kL2BrpEMNM

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: Sonin condition, general fractional integrals, general fractional derivatives, sequential general fractional derivatives, fundamental theorems of fractional calculus, convolution series, convolution Taylor formula, convolution Taylor series.

Abstract: In this talk, we start with a discussion of origins of the general fractional integrals and derivatives with the Sonin kernels in the works by Abel and Sonin. Then some recent results regarding properties of the general Fractional Calculus operators with the Sonin kernels are presented. In particular, the first and the second fundamental theorems of Fractional Calculus for the general fractional derivatives, the regularized general fractional derivatives, and the sequential general fractional derivatives are formulated. As an application, we discuss the convolution series that are a far-reaching generalization of the power law series as well as a representation of functions in form of the convolution Taylor series. Another application is the generalized convolution Taylor formula that contains a convolution polynomial and a remainder in terms of the general fractional integrals and the general fractional sequential derivatives. Finally, we provide a survey of some other recent results devoted to the general Fractional Calculus operators with the Sonin kernels and their applications.

Biography: Dr. Yuri Luchko is a Full Professor at the Faculty of Mathematics - Physics - Chemistry of the Berlin University of Applied Sciences and Technology in Germany. He studied Mathematics at the Belarussian State University in Minsk and received his PhD degree from the same University in 1993. In 1994, Yuri Luchko got a postdoc position at the Free University of Berlin, Germany, under supervision of Prof. Rudolf Gorenflo and stayed there for six years. From 2000 to 2006, he was a scientific researcher at the University in Frankfurt (Oder), Germany. In 2006, Dr. Yuri Luchko got a professorship at the Technical University of Applied Sciences Berlin, Germany. The main field of his research is Applied Mathematics with a special focus on Fractional Calculus and its applications. Yuri Luchko published about two hundred papers in international peer-reviewed scientific journals and about twenty books and books chapters as author or editor. He is an associate editor of the international journal "Fractional Calculus and Applied Analysis" and editor of several other reputable mathematical journals including ZAA (Zeitschrift für Analysis und ihre Anwendungen).

Bibliography

[1] Y. Luchko. *General Fractional Integrals and Derivatives with the Sonine Kernels* Mathematics 2021, vol. 9(6), 594.

[2] Y. Luchko. *General Fractional Integrals and Derivatives of Arbitrary Order* Symmetry 2021, vol. 13(5), 755.

[3] Y. Luchko. *Convolution series and the generalized convolution Taylor formula* Fract. Calc. Appl. Anal. 2022, vol. 25, 207-228.

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10 May 2024
14:30-15:30
CEST

An Introduction to Fractional Calculus

Francesco Mainardi
University of Bologna and INFN, Italy

Time: 14:30 - 15:30 CEST (Rome / Paris)
Date: 10 May 2024
YouTube: https://youtu.be/YxaU1HmXJcU

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy
Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: fractional calculus, Mittag-Leffler functions, Wright functions

Abstract: We introduce the linear operators of fractional integration and fractional differentiation in the framework of the so called fractional calculus (FC). Our approach is essentially based on an integral formulation of fractional calculus acting on sufficiently well behaved functions defined in $\mathbb{R}$ that include the Riemann-Liouville, Caputo and Riesz-Feller approaches with the related special functions. The list of applications of FC is huge and includes, just to cite a few, Visco-elasticity, Electrical Circuits, Control theory, intermediate phenomena between Diffusion and Wave propagation, Biology, Bioengineering, Image processing, Finance, Stochastic processes.

Biography: Francesco MAINARDI is retired professor of Mathematical Physics from the University of Bologna (since November 2013) where he has taught this course since 40 years. Even if retired, he continues to carry out research activity. His fields of research concern several topics of applied mathematics, including linear viscoelasticity, diffusion and wave problems, asymptotic methods, integral transforms, special functions, fractional calculus and non-Gaussian stochastic processes. At present his H-index is > 70.

For a full biography, list of references on author's papers and books see:
Home Page: http://www.fracalmo.org/mainardi/index.htm
Profile: http://scholar.google.com/scholar?hl=en&lr=&q=f+mainardi

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[1] R. Gorenflo, A.A Kilbas, F. Mainardi and S.V. Rogosin, *Mittag-Leffler Functions. Related Topics and Applications*, Springer, Berlin (2020), Second Edition.

[2] F. Mainardi, *Fractional Calculus and Waves in Linear Viscoelastity*, World Scientific, Singapore (2022), Second Edition.

Classes of I- and bar-H- special functions related to Fractional Calculus and generalized fractional integrals

17 May 2024
14:30-15:30
CEST

Virginia Kiryakova
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

Time: 14:30 - 15:30 CEST (Rome / Paris)
Date: 17 May 2024
YouTube: https://youtu.be/SHBgjrNh4A0

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy
Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: special functions, fractional calculus, integral operators

Abstract: Recently, the interest of authors like Gerhold, Garra, Polito, Mainardi, Rogosin, Garrappa, Gorska and others has been attracted to the so-called Le Roy function that happened to appear almost same time as the famous Mittag-Leffler function and for almost same goals, initially not associated with Fractional Calculus (FC). They have introduced and studied its extension as a hybrid between the Mittag-Leffler and Le Roy functions, depending on fractional parameters both as arguments in the Gamma function and as its fractional power. Next came a series of multi-index generalizations as new classes of Special Functions (SF) related to FC (Kiryakova, Paneva-Konovska, Rogosin, Dubatovskaya: [2], [4], [5], [6], [7]).

The idea to relate the Le Roy type multi-index functions to the not so-popular I-functions of Rathie and bar-H-functions of Inayat-Hussain, as further extensions of the Fox H-functions, happened to be very fruitful. It happened that not only the Le Roy type functions belong to their class, but also functions appearing by needs in Physics and Statistics (as Feynman integrals) and some well known mathematical functions as polylogarithms, Riemann Zeta-functions, its extensions, etc., see [8].

First, this led us to introduce and study the analytical properties of a new entire function $\widetilde{{}_p\Psi_q}$ extending the Fox-Wright function ${}_p\Psi_q$, thus to encompass yet more of the SF related to Classical and Fractional Calculus, as previously listed in [4].

Then, it is the problem to identify the integral (and possibly differential) operators for which such SF as of Le Roy type play the role of eigen-functions, similarly like the exponential / trigonometric functions appear as eigen-functions of the operators of integer order Calculus, and the Mittag-Leffler type functions - for the R-L and Erdélyi-Kober operators of FC. The Gelfond-Leontiev theory for generalized integrations and differentiation (cf. [3]) helped to resolve the case, first in form of power series, and then also as integral operators with I-function kernels, [8]. The next step is to introduce integral operators with more general $I^{m,0}_{m,m}$ kernels, for which we prove to satisfy the semi-group property and other axioms of FC. These appear as further extensions of our operators of Generalized Fractional Calculus [1], with $H^{m,0}_{m,m}$ kernels, and are also compositions of $m \geq 1$ generalized Erdélyi-Kober integrals.

The talk is based on recent results and publications by V. Kiryakova and J. Paneva-Konovska, partly collaborated also with S. Rogosin and M. Dubatovskaya.

Biography: Virginia Kiryakova is a Bulgarian mathematician, Professor Emeritus in Institute of Mathematics and Informatics at Bulgarian Academy of Sciences. Her research results in more than 140 publications for about 50 years are in fractional calculus, special functions and integral transforms, and on the history of fractional calculus, author of the monograph "Generalized Fractional Calculus and Applications", Longman-J. Wiley, 1994. These works are cited more than 7000 times and she is in the list of the top 2% of scientists worldwide according to the Standford University ranking. Aside from her role as Editor-in-Chief of the top-ranked specialized journal "Fractional Calculus and Applied Analysis" (Springer), for long years she has a lot of organizational activities in scientific and social organizations, committees, etc. and holds some national and international prizes.

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[1] V. Kiryakova. *Generalized Fractional Calculus and Applications*, Longman - J. Wiley, Harlow- New York (1994).

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Analysis and Modeling of Non-local Eddy Diffusivity in Turbulent Flows

24 May
14:30-15:30
CEST

Fujihiro Hamba
Institute of Industrial Science, The University of Tokyo

Time: 14:30 - 15:30 CEST (Rome / Paris)
Date: 24 May 2024
YouTube: https://youtu.be/v1ZAyn2urqE

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: turbulence model, non-local eddy diffusivity

Abstract: Local eddy viscosity and diffusivity models are widely used to understand and predict the mean velocity and mean scalar quantities in turbulent flows. In the eddy diffusivity model, the turbulent scalar flux at a point is assumed to be proportional to the mean scalar gradient at the same point. However, this local approximation is not always valid for real turbulent flow because the length scale of turbulence is often as large as that of the mean-field variation. In this talk, we present a non-local approach in which the scalar flux is expressed as spatial and time integrals of the mean scalar gradient. We first introduce our previous work on the analysis of non-local eddy diffusivity and viscosity using the Green's function [1-3]. Profiles of the non-local eddy diffusivity were evaluated and the non-local expressions were validated with a direct numerical simulation (DNS) of turbulent channel flow. In some cases, the local eddy diffusivity model overpredicted the scalar flux, and the non-local expression accurately estimated it. We then present recent attempts to model the non-local eddy diffusivity from the viewpoint of statistical theory of turbulence [4,5]. The behavior of the non-local eddy diffusivity was carefully examined through a DNS of homogeneous isotropic turbulence with an inhomogeneous mean scalar. A model expression was proposed using the mean Green's function and the two-point velocity correlation. The idea of energy density in scale space [6] was also used to allow the model to be applied to inhomogeneous wall-bounded flows.

Biography: Fujihiro Hamba is a full professor in fluid physics at the Institute of Industrial Science, the University of Tokyo in Japan. He studied physics and received his PhD degree from the Graduate School of Science, the University of Tokyo in 1990. His research interests are in physics and modeling of inhomogeneous turbulence, including non-local transport of turbulence, Reynolds-averaged Navier-Stokes model and large eddy simulation, and dynamo effects in magnetohydrodynamic turbulence.

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Fractional diffusion models of nonlocal turbulent transport in plasmas and fluids

31 May
15:00-16:00
CEST

Diego del-Castillo-Negrete

Oak Ridge National Laboratory, USA

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 31 May 2024
YouTube: https://youtu.be/vLZC3pN8pnw

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: fractional diffusion, nonlocal transport, anomalous transport, plasma physics, fusion plasmas, turbulent transport, Feynman–Kac

Abstract: There is growing experimental, numerical, and theoretical evidence of cases where local (advection-diffusion) transport models fail to describe anomalous transport (e.g., super-diffusive and sub-diffusive transport). To overcome this limitation, nonlocal models introduce nonlocal flux-gradient relations and formulate transport using integrodifferential operators in general and fractional diffusion operators in particular. In this lecture we present an overview of fractional diffusion models of nonlocal transport with emphasis on applications to plasmas and fluids. In the first part, a brief review of the statistical foundations of general nonlocal models will be presented. Following that, we will discuss experimental and numerical evidence of nonlocal transport in magnetically confined fusion plasmas and fluids, along with effective fractional diffusion models describing these phenomena. The second part of the lecture will be devoted to a recently proposed probabilistic method to solve the initial value problem and the exit-time problem of fractional diffusion equations. The method is based on the Feynman-Kac formula and reduces the computation to the evaluation of expectations (i.e., numerical quadratures), bypassing the need of sampling stochastic trajectories (as in the case of particle-based methods) or the need to deal with dense, non-sparse matrices (as in the case of continuum methods). A description of the method will be presented along with applications.

Biography: Diego del-Castillo-Negrete is a Distinguished Scientist in the Theory and Modeling Group of the Fusion Energy Division at the Oak Ridge National Laboratory in USA. He holds a Ph.D. in Physics from the University of Texas at Austin (1994). Before joining ORNL in 2000, he worked in the Theoretical Division of Los Alamos National Laboratory (1998-2000), and the Scripps Institution of Oceanography at the University of California San Diego (1994-1998). His research interests span a wide spectrum of topics in plasma physics, applied mathematics, nonlinear dynamics, computational physics, and machine learning.

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Fractional-Order Large-Eddy Simulation of Turbulence

07 June
15:00-16:00
CEST

Mohsen Zayernouri
Associate Professor
Department of Mechanical Engineering
Department of Statistics and Probability
Michigan State University (MSU), East Lansing, MI 48824

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 07 June 2024
YouTube: https://youtu.be/Me54VO60jTA

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: anomalous subgrid dynamics, multi-scale modelling

Abstract: Turbulence remembers and is fundamentally nonlocal. Such a longing portrait of turbulence originates from the delineation of coherent structures/motions, being spatially spotty, giving rise to interestingly anomalous spatio-temporal fluctuating signals. The statistical anomalies in such stochastic fields emerge as: sharp peaks, heavy-skirts of power-law form, long-range correlations, and skewed distributions, which scientifically manifest the non-Markovian/non-Fickian nature of turbulence at small scales. Such physical-statistical evidence highlights that 'nonlocal features' and 'global inertial interactions' cannot be ruled out in turbulence physics. On a whole different (computational) level and in addition to the aforementioned picture, the very act of filtering the Navier-Stokes and the energy/scalar equations in the large eddy simulations (LES) would make the existing hidden nonlocality in the subgrid dynamics even more pronounced, to which it induces an immiscibly mixed physical-computational nonlocal character.

This urges the development of new LES modeling paradigms in addition to novel statistical measures that can meticulously extract, pin-down, and highlight the nonlocal character of turbulence (even in the most canonical flows e.g., homogeneous isotropic turbulence) and their absence in the common/classic turbulence modeling practice. We start from the filtered Boltzmann kinetic transport equations and model the corresponding equilibrium distribution functions (for both the fluid and scalar particles) with stable heavy-tailed distributions to address and incorporate the anomalous features at small scales. Next, we derive a new class of fractional-order and tempered Laplacian models for the divergence of subgrid-scale stresses, naturally emerging as the underlying subgrid-scale (SGS) LES models. We subsequently carry out the corresponding a priori and a posteriori tests to examine the performance of each fractional SGS model. Our proposed dynamic LES modeling approach exhibits promising capabilities to effectively model and incorporate nonlocalities on the fly in the very LES (VLES) as well as the LES inertial sub-ranges. This novel LES modeling paradigm can be imperative for cost-efficient nonlocal turbulence modeling e.g., in meteorological and environmental applications. A survey of our developments in this paradigm of research is given in [1-8].

Biography: Mohsen Zayernouri is an associate professor at MSU and the director of FMATH group https://fmath.msu.edu. He obtained his 2nd PhD in Applied Mathematics at Brown University back-to-back after obtaining his 1st PhD in Mechanical Engineering at the University of Utah. He received the AFOSR Young Investigator Program (YIP) award in 2017 in addition to the ARO YIP award in 2019. He served as the MSU-PI in the ARO-MURI on Fractional PDEs for Conservation Laws and Beyond. He has also received multiple other awards from the US National Science Foundation, the US Army Research Lab (ARL) on relevant topics, including: statistical learning, uncertainty quantification, nonlocal turbulence modeling, and multi-scale (MD-to-DDD-to-Continuum) nonlocal material failure modeling.

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Subgrid-scale modeling with memory using the Mori—Zwanzig formalism and variational multiscale method

07 June
16:00-17:00
CEST

Eric Parish

Sandia National Laboratories

Time: 16:00 - 17:00 CEST (Rome / Paris)
Date: 07 June 2024
YouTube: https://youtu.be/XS-0btf7AP0

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: Mori–Zwanzig, turbulence, memory

Abstract: This talk discusses the development of subgrid-scale models for turbulent flows using the Mori—Zwanzig (MZ) formalism [1,2]. Originating from irreversible statistical mechanics and re-formulated by Chorin et al. in the early 2000s [3], the MZ formalism provides a mathematical procedure for the development of coarse-grained models of complex systems, such as turbulence, that lack scale separation. The formalism operates by partitioning the state of a dynamical system into a resolved "coarse-grained" component and an unresolved component. The impact of the unresolved scales on the resolved scales is then re-cast as a convolutional "memory" integral that depends only on the resolved scales. While exact computation of this memory integral is intractable, it provides a starting point for the systematic construction of subgrid-scale models. In this talk, we outline the development of MZ-based subgrid-scale models for coarse-grained simulations of turbulence. We introduce the Mori—Zwanzig formalism, and then provide a formulation for developing MZ-based subgrid-scale models for coarse-grained PDEs by combining MZ with the variational multiscale method (VMS) [4]. We introduce several approximations to the memory term and compare them with established techniques [4,5,6]. In doing so, we establish links between existing stabilization techniques and memory effects. Lastly, numerical results are shown for a variety of canonical turbulent flows.

Biography: Eric Parish is a member of technical staff at Sandia National Laboratories. He received his PhD from the University of Michigan in 2018 and was the John von Neumann postdoctoral fellow at Sandia from 2018-2020. At Sandia, he works broadly in the fields of turbulence modeling, model reduction, and scientific machine learning. His present work is primarily targeted at developing an improved modeling capability for hypersonic turbulent flows.

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Hyperdiffusion of dust particles in a turbulent fusion plasma

14 June
15:00-16:00
CEST

Federico Nespoli

Princeton Plasma Physics Laboratory

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 14 June 2024
YouTube: Recording failure due to a technical issue

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: plasma turbulence, dust, diffusive processes

Abstract: Tokamaks and stellarators are a promising candidate for future nuclear fusion reactors, where a hot, turbulent plasma is confined by a toroidal magnetic field. Due to the interaction of the plasma with the solid walls of the reactor, dust is produced and enters the plasma, polluting it. Furthermore, in recent experiments dust is introduced deliberately into the fusion plasma, for a range of different applications including turbulence control and wall conditioning. The understanding of the dust dynamics in a turbulent fusion plasma is therefore an active topic of research.

In this talk, we investigate the effect of plasma turbulence on the dust dynamics, using the ad hoc developed Dust Injector Simulator code [1] in combination with a turbulent plasma background computed by the TOKAMK3X code [2] in tokamak geometry. The plasma turbulence is observed to scatter the dust particles, exhibiting a hyperdiffusive regime in all cases. The amplitude of the turbulent spread of the trajectories is characterized in terms of the adimensional parameters defining the system, namely the Stokes number and the Kubo number. The results are interpreted with a simple analytical model.

Biography: Federico Nespoli is a staff research physicist at Princeton Plasma Physics Laboratory. He is working on both experiments and simulations on the effect of impurities on the plasma edge in stellarators and tokamaks. Federico earned his bachelor and master in Physics at Universita' degli Studi di Milano in Italy, and his PhD from Ecole Federale Polytechnique Lausanne in Switzerland. After that he worked as a postdoctoral researcher at Aix-Marseille University in France and finally moved to Princeton Plasma Physics Laboratory, USA.

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21 June
15:00-16:00
CEST

Phase transitions in the fractional three-dimensional Navier-Stokes equations

Daniel W. Boutros
University of Cambridge

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 21 June 2024
YouTube: https://youtu.be/05mHFr0MZ6s

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: fractional Navier–Stokes equations, regularity criteria, energy balance, phase transition, hypodissipation

Abstract: The fractional Navier-Stokes equations are a generalisation of the Navier-Stokes equations, in which the Laplacian in the viscous term is replaced by a fractional Laplacian. We study the functional properties of solutions to these equations as the exponent of the fractional Laplacian (which we will refer to as s) goes to zero (while keeping the viscosity fixed), instead of studying the conventional inviscid limit in which the viscosity is sent to zero. We find four critical values of the exponent, at which a qualitative change in behaviour of the solutions occurs. Such a change is referred to as a phase transition. In particular, we establish three results: i) a continuation criterion for strong solutions for $s > 1/3$ ii) an equation of local energy balance if $s \geq 3/4$ iii) an infinite hierarchy of regularity estimates on higher order derivatives for Leray-Hopf solutions if $s > 5/6$. This paper [1] is joint work with John D. Gibbon (Imperial College London).

Biography: Daniel W. Boutros is a PhD student at the Department of Applied Mathematics and Theoretical Physics at the University of Cambridge, under the supervision of Professor Edriss S. Titi. He received his Bachelor's degrees from the University of Groningen, and his Master's degree from the University of Cambridge. His research interests lie in the analysis of partial differential equations, in particular in connection with hydrodynamic turbulence theory and geophysical fluid mechanics. In particular, he has worked on the mathematical analysis of large-scale oceanic dynamics, subgrid-scale turbulence modelling and the study of turbulent wall-bounded flows.

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Fractional Navier-Stokes equations

21 June
16:00-17:00
CEST

Pavan Pranjivan Mehta
SISSA, International School of Advanced Studies, Italy

Time: 16:00 - 17:00 CEST (Rome / Paris)
Date: 21 June 2024
YouTube: https://youtu.be/d-sPRoG1Tm0

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: fractional Navier–Stokes equations, fractional Cauchy equations, fractional continuity equation, fractional Reynolds-averaged Navier—Stokes equations, turbulence

Abstract: Turbulence is a non-local and multi-scale phenomenon. Resolving all scales implies non-locality is addressed implicitly. However, if spatially or temporally averaged fields are considered for computational feasibility, then addressing non-locality explicitly becomes important as a result of missing information of all scales.

Our previous work involved constructing fractional closure model [1,2] for Reynolds-averaged Navier–Stokes equations, which are temporally averaged. In [2] "fractional stress-strain hypothesis" was introduced using a a variable-order (spatially-dependent) Caputo fractional derivative. Indeed, it addresses the amalgamation of local and non-local effects [1, 2]. The results of two-sided model [2] were very encouraging, where, we find a power-law behaviour of fractional order akin to logarithmic regime for velocity and also a law of wake. Further, we investigated tempered fractional definitions in [2]. Since turbulence is a decay process, it led to define a "horizon of non-local interactions" in [2].

Despite the success, one overwhelming question remains, "how do we derive a fractional conservation law from first principles?" Thus, in this talk, I shall introduce the recently developed control volume approach in [3] to derive fractional conservation law from first principles. The first step was to extend the fractional vector calculus developed in [4] to two-sided operators [3]. Subsequently, derive the fractional analogue of Reynolds transport theorem [3]. By virtue of this theorem, we derive the "fractional continuity" and "fractional Cauchy equations", which follows conservation of mass and momentum, respectively [3]. The stress tensor of fractional Cauchy equation is treated with a fractional stress-strain relationship (which was developed in [2]) to get to the final form of "fractional Navier–Stokes equations".

Biography: Pavan Pranjivan Mehta is a PhD student within the mathematics area of SISSA, Italy. He hold's two Master's degree, namely, Thermal and Fluid Engineering from University of Manchester, UK and Applied Math from Brown University, USA; with an undergrad in Aeronautical Engineering. He has held research positions in France, USA, UK and India; with an internship at Airbus Group and visiting researcher at Newton Institute, Cambridge for two scientific programs, namely, turbulence and fractional differential equations. Pavan's organisational activities for fractional calculus, includes mini-symposiums at prestigious conferences and seminar series, such as JINX seminar's at Newton Institute and weekly seminars at SISSA. His current research interests are non-local turbulence modeling and numerical methods for fractional PDE.

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Efficient Algorithms for Computing Fractional Integrals

28 Jun 2024
15:00-16:00
CEST

Kai Diethelm
Technical University of Applied Sciences Würzburg-Schweinfurt

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 28 June 2024
YouTube: https://youtu.be/AZbn-spD6nU

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: Riemann–Liouville integral, numerical method, fast solution, diffusive representation, sum-of-exponentials method, kernel compression

Abstract: Because of the inherent hereditary properties of fractional operators, the numerical evaluation of the Riemann-Liouville integral of some given vector-valued function at a large number of grid points is a computationally expensive and memory-intesive task when classical algorithms are used [5, 6, 7]. Applying FFT-based summation techniques to the classical methods significantly reduces the run time but not the memory demands [11, 12]. In recent years, we have thus witnessed the development of a growing number of algorithms aimed at reducing both run time and memory requirements. These methods typically use kernel compression techniques, sum-of-exponentials approximations to the kernel function, or nonclassical representations of the fractional integral operator, cf., e.g., [4, 2, 3, 1, 8, 9, 10]. In this talk, we present a common framework, based on the concept of diffusive representations of fractional operators, under which we can subsume many seemingly different types of such novel methods. This more abstract perspective allows to obtain additional insight and a better understanding of how the algorithms behave. This is particularly significant because these methods are at the core of efficient schemes for solving fractional differential equations.

Biography: Kai Diethelm is Professor for Mathematics and Applied Computer Science at the Technical University of Applied Sciences Würzburg-Schweinfurt, Germany, where he leads the Scientific Computing Laboratory. He holds a Diploma in Mathematics from TU Braunschweig and a PhD in Computer Science from the University of Hildesheim. His main research interests are approximation theory and its applications, analytical and numerical aspects of fractional calculus (especially, fractional ordinary differential equations), and high performance computing.

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Stochastic processes on infinite-dimensional spaces and fractional operators

05 July
15:00-16:00
CEST

Luisa Beghin
Dep. Statistical Sciences / Sapienza University, Rome

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 05 July 2024
YouTube: https://youtu.be/zJewi7Uy6oc

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: fractional operators, grey Brownian motions, incomplete gamma function, Tricomi hypergeometric function.

Abstract: We present and analyze some stochastic processes defined in analogy with fractional Brownian motion, in infinite-dimensional white or grey-noise spaces. By means of Riemann-Liouville, Hadamard-type or general fractional operators, we extend grey Brownian motion (see e.g. [1], [2]), providing new models for anomalous diffusions. In particular, we consider the non-Gaussian process defined by means of the so-called incomplete gamma measure ([3]). Moreover, we present a Gaussian class of processes linked to the Hadamard derivatives or integrals ([4]) and to generalized fractional operators with Sonine-pair kernels ([5]).

Biography: Luisa Beghin is a full professor in Probability and Mathematical Statistics at Sapienza University in Rome. She obtained a PhD. in Methodological Statistics from Sapienza University in 2000. Her current research interests are primarily in stochastic processes linked to fractional calculus and non-local operators, Lévy processes, subordination and time-change theory.

Bibliography

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12 July 2024
15:00–16:00
CEST

Fundamental solution of the space-time fractional diffusion equation using Monte Carlo simulations of CTRWs

Enrico Scalas

Department of Statistical Sciences / Sapienza University of Rome, Italy

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 12 July 2024
YouTube: https://youtu.be/5x3k3Rxw_K0

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: space-time fractional diffusion equation, continuous-time random walks, Monte Carlo simulations

Abstract: I present a numerical method for the Monte Carlo simulation of uncoupled continuous-time random walks with a Lévy α-stable distribution of jumps in space and a Mittag–Leffler distribution of waiting times, and apply it to the stochastic solution of the Cauchy problem for a partial differential equation with fractional derivatives both in space and in time. The one-parameter Mittag–Leffler function is the natural survival probability leading to time-fractional diffusion equations. Transformation methods for Mittag–Leffler random variables were found later than the well-known transformation method by Chambers, Mallows, and Stuck for Lévy α–stable random variables and they are now widely used. Combining the two methods, one obtains an accurate approximation of space– and time–fractional diffusion processes almost as easy and fast to compute as for standard diffusion processes. If time allows, I will discuss directions for further research.
This was joint work with Daniel Fulger and Guido Germano [1].
I delivered a five-hour course on this topic at the on–line UG LMS Summer School held in Swansea in 2021 [2].

Biography: Enrico Scalas is professor of probability and mathematical statistics at the Department of Statistical Sciences of Sapienza, University of Rome. He is co-author of more than 140 publications including two monographs and an edited book. If you like bibliometry, you'll be delighted to know that his works received more than 11000 citations and his h index is 43 (according to Google Scholar). Otherwise, you may just be interested in the fact that his research concerns the foundations of economics, finance and statistical physics and, for this reason, he became a probabilist working on various stochastic processes often with fractional flavour. For the time being, a rather complete list of his publications up to 2022 can be found at:
`https://www.sussex.ac.uk/profiles/330303/publications`. More recent publications are listed at
`https://corsidilaurea.uniroma1.it/it/users/enricoscalasuniroma1it`, following the link *Profilo Research - Pubblicazioni IRIS*.

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19 July 2024
15:00-16:00
CEST

Fractional Brownian motion with random as well as time- and space-dependent Hurst exponent

Ralf Metzler

Institute of Physics & Astronomy, University of Potsdam, 14476 Potsdam, Germany

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 19 July 2024
YouTube: https://youtu.be/c8pMb3hXDEQ

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: anomalous diffusion, long-range dependence, fractional Brownian motion

Abstract: Stochastic processes with long-range dependent correlations naturally emerge in many systems when all degrees of freedom are integrated out, apart from the (tracer) particle of interest. Examples are the dynamics of a labelled monomer in Rouse-type polymer chains or test particles in viscoelastic solutions. In non-equilibrium situations, the resulting overdamped dynamics corresponds to fractional Brownian motion (FBM).

This talk will first address disordered systems, when the observed displacement probability density turns out to be non-Gaussian. For the descriotion of such situations a random-diffusivity (doubly-stochastic) FBM model [1,2] will be introduced and shown that the precise dynamics encoded in this model delicately depends on its precise mathematical formulation.

This talk will then introduce Lévy's formulation of FBM via a Riemann-Liouville fractional integral. It will be discussed how time- and space-dependent scaling (Hurst) exponents and diffusion coefficients can be introduced in this model, and which dynamic behaviour emerges [3,4]. Again, the specific formulation of the model results in significantly different dynamical behaviours.

Biography: After receiving his doctorate at the University of Ulm, Ralf Metzler went to Tel Aviv University for a postdoc with Prof Yossi Klafter. He then continued for a second postdoc with Prof Mehran Kardar at MIT before his first faculty appointment as assistant professor at the Nordic Institute for Theoretical Physics (NORDITA) in Copenhagen. Ralf then became associate professor and Canada Research Chair for Biological Physics at the University of Ottawa. In 2007 Ralf was appointed as professor at the Technical University of Munich, and since 2011 he holds the chair for Theoretical Physics at the University of Potsdam. From 2010-2015 Ralf was also a Finland Distinguished Professor at Tampere University of Technology. Among others, Ralf was awarded the 2017 SigmaPhi prize and an Alexander von Humboldt Polish Honorary Research Scholarship (2018-2022). Ralf's work focuses on statistical and biological physics, in particular, on stochastic processes and their applications.

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W. Wang, A. G. Cherstvy, A. V. Chechkin, S. Thapa, F. Seno, X. Liu, and R. Metzler. *Fractional Brownian motion with random diffusivity: emerging residual nonergodicity below the correlation time.* J. Phys. A **53**, 474001 (2020).

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19 July 2024
16:00-17:00
CEST

Fractional Boundary Value Problems: Results and Applications

Mirko D'Ovidio

Dept. of Basic and Applied Sciences for Engeenering / Sapienza University of Rome

Time: 16:00 - 17:00 CEST (Rome / Paris)
Date: 19 July 2024
YouTube: https://youtu.be/e1FsDXNKp_o

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: sticky Brownian motions, nonlocal dynamic conditions, time changes, metric graphs

Abstract: Sticky diffusion processes spend finite time (and finite mean time) on a lower-dimensional boundary. Once the process hits the boundary, then it starts again after a random amount of time. While on the boundary it can stay or move according to dynamics that are different from those in the interior. Such processes may be characterized by a time derivative appearing in the boundary condition for the governing problem. We use time changes obtained by right-inverses of suitable processes in order to describe fractional (or nonlocal in general) sticky conditions and the associated boundary behaviours. We obtain that fractional boundary value problems (involving fractional dynamic boundary conditions) lead to sticky diffusions spending an infinite mean time (and finite time) on a lower-dimensional boundary. Such a behaviour can be associated with a trap effect in the macroscopic point of view.

For the nonocal time boundary conditions, we first discuss the apparently simple case of the half line with boundary of zero Lebesgue measure. In this case we present some applications concerned with motions on metric graphs. Such results turn out to be instructive for the general case of boundary with positive (finite) Borel measures. In this regard, we provide some results on open, connected and non-empty sets with smooth boundaries and describe possible applications involving motions on irregular domains, fractals for instance.

We briefly discuss also nonlocal space conditions on the boundary and the associated processes. The underlying dynamics can be related with the stochastic resetting.

The talk is based on the works listed below in the References.

Biography: Mirko D'Ovidio is associate professor in Probability and Mathematical Statistics at Sapienza University of Rome. He works on the connections between stochastic processes and PDEs, time changes and boundary value problems, nonlocal operators and irregular domains.

Bibliography

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26 July
15:00-16:00
CEST

Nonlinear Fokker–Planck equations with fractional Laplacian and McKean–Vlasov SDEs with Lévy–Noise

Michael Röckner
Bielefeld University

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 26 July 2024
YouTube: https://youtu.be/ODyxvSv3jAA

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: nonlinear Fokker–Planck equation, fractional Laplacian, nonlinear Markov process, McKean–Vlasov SDE, Levy noise

Abstract: This talk is concerned with the existence of mild solutions to nonlinear Fokker–Planck equations with fractional Laplace operator $(-\Delta)^s$ for $s \in (\frac{1}{2}, 1)$. The uniqueness of Schwartz distributional solutions is also proved under suitable assumptions on diffusion and drift terms. As applications, weak existence and uniqueness of soultions to McKean–Vlasov equations with Lévy–Noise, as well as the Markov property for their laws are proved.

Biography: Michael Röckner is a Professor of Mathematics at Bielefeld University. He has had previous professorial positions at Purdue University and the University of Bonn and was a Reader at the University of Edinburgh. He received his Doctor Degree and his Habilitation from Bielefeld University. He won the Sir Edmund Whittaker Memorial Prize, the Heinz Mayer Leibnitz Prize and the Max Planck Research Prize. In 2017 he was awarded an honorary Dr. of Science degree by the University of Swansea. He is a Distinguished Visiting Professor at the Academy of Mathematics and Systems Science in Beijing and won the Award for International Cooperation of the Chinese Academy of Sciences in 2023. On June 24, 2024, he received the International Science and Technology Cooperation Award of the People's Republic of China. He is a member of the Academia Europaea, the Academy of Science and Literature, Mainz, and a foreign honorary member of the Romanian Academy of Sciences.

He was the President of the German Mathematical Society (DMV) and the Director of the Centre for Interdisciplinary Reseach (ZiF) in Bielefeld. He served as a panel chair for the Advanced Grants of the European Research Council, as a member of the Selection Committee for the Senior Humboldt Awards, as a member of the Senate's Commission for Collaborative Research Centers of the German Science Foundation (DFG) and for 11 years as the Dekan of Bielefeld University's Faculty of Mathematics.

Apart from six books he has published close to 400 research articles, supervised 31 PhD students and was host for 10 Humboldt scholars. His main field of research is Stochastic Analysis.

Bibliography

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02 Aug.
15:00-16:00
CEST

Semi-Markov processes, time-changes and non-local equations

Bruno Toaldo
Dept. of Mathematics – University of Turin

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 02 August 2024
YouTube: https://youtu.be/NIpnfbYAodQ

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: semi-Markov processes, non-local equations, continuous time random walk limits

Abstract: I speak about the theory of semi-Markov processes in connection with non-local equations. Therefore, I first introduce semi-Markov processes speaking about definitions in the sense of Gihman and Skorohod [1] and in the sense of Lévy (and Harlamov [2]) and then, I will explain the time-change technique in connection with semi-Markov property, and also CTRW limit processes [3]. Using suitable time-changes of Markov processes it is possible to establish a connection with several non-local equations, in time and space, with expectation of semi-Markov processes. I will speak about the classical time-change with independent inverse subordinators and also about more recent and new directions, e.g., time-change with additive components [4] and with undershooting of subordinators [5].

Biography: Bruno Toaldo is an Associate Professor of Probability at the Department of Mathematics "Giuseppe Peano", University of Turin. Born on 1985. He earned his Ph.D from Sapienza - University of Rome in 2013. Toaldo's academic journey includes a tenure-track (Univ of Turin) and junior (Univ of Naples) researcher positions, along with a postdoctoral fellowship (Sapienza – Univ of Rome), primarily focusing on probability theory, nonlocal operators, and semi-Markov processes. He has served in numerous scientific and editorial roles. His research interests extend to modeling aspects related to anomalous transport, diffusion, and applications in mathematical finance and neuronal modeling.

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09 Aug.
15:00
CEST

Discrete-Time General Fractional Calculus

Anatoly N. Kochubei
Institute of Mathematics, National Academy of Sciences of Ukraine

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 9 Aug 2024
YouTube: https://youtu.be/LEtO2sxJM1c

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: general fractional calculus, Stieltjes functions, fractional difference equations, resolvent families, Poisson transform

Abstract: Discrete-time fractional calculus is an important branch of fractional analysis actively developed in the last decades. On the one hand, this kind of analysis generates a lot of interesting mathematical problems. On the other, there are many applications to real-world problems, such as new encryption methods, fractional discrete-time neural networks and others. Up to now, all this was built on appropriate analogs of the classical Riemann–Liouville and Caputo fractional derivatives.

Meanwhile for continuous-time fractional calculus, there exists an extension called General Fractional Calculus (GFC) introduced in [1]. In GFC, the counterpart of the fractional time derivative is a differential-convolution operator whose integral kernel satisfies some additional conditions, under which the Cauchy problem for the corresponding time-fractional equation is not only well-posed, but has properties similar to those of classical evolution equations of mathematical physics.

In this work (joint with Alexandra Antoniouk), we develop the GFC approach for the discrete-time fractional calculus. We follow the idea of Lizama [2] who proposed to define discrete-time operations as the Poisson transforms of their continuous-time counterparts. In particular, we define within GFC the appropriate resolvent families and use them to solve the discrete-time Cauchy problem with an appropriate analog of the Caputo fractional derivative.

Biography: Anatoly N. Kochubei (born in 1949) is Department Head at the Institute of Mathematics of the National Academy of Sciences of Ukraine. His research interests include fractional calculus, non-Archimedean analysis, partial differential equations, operator theory, and mathematical physics.

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Rates of convergence of CTRWs to generalised fractional evolutions

16 Aug.
15:00
CEST

Vassili Kolokoltsov

Department of Statistics, University of Warwick, Coventry, United Kingdom

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 16 August 2024
YouTube: https://youtu.be/cNgHmBIiijM

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: rates of convergence, continuous-time random walks, scaling limits, fractional evolutions, functional central limit theorem with stable processes, Levy flights and Levy walks, domains of quasi-attraction

Abstract: The talk will be devoted to three new interlinked directions of research:

1. Rates of convergence in the functional CLT with stable limits (including domains of quasi-attraction: for large, but not too large n);
2. Rates of convergence of CTRWs (continuous time random walks) to fractional evolutions;
3. Rates of convergence for the scaling limits of Lévy flights and Lévy walks.

Biography: Vassili N. Kolokoltsov is Professor at Lomonosov Moscow State University and Major researcher at National Research University Higher School of Economics Moscow, Russia. He is also Professor Emeritus at the University of Warwick, UK. Professor Kolokoltsov got his PhD in mathematics from Moscow State University in 1984 and the full Doctor of Science in mathematics and physics from the Steklov Math Institute of the Russian Academy of Science in 1993. Vassili Kolokoltsov published 12 monographs and over 200 research papers in various domains of pure and applied mathematics. His research interests cover a wide spectrum of mathematical disciplines including probability, stochastic analysis, optimisation theory, game theory, differential equations, fractional calculus, functional analysis, mathematical physics, quantum control and games, mathematical modelling of socio-economic, ecological and financial processes. During his scientific career, Professor Kolokoltsov visited and worked in various research centres in Europe and America, including the UK, Germany, France and Mexico, and took part in numerous international conferences as speaker, organiser, or member of the program committee. He also has been serving as an editor in several mathematics journals including Fractional Calculus and Applied Analysis (FCAA) and Dynamic Games and Applications (DGA).

Bibliography

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[2] Vassili N. Kolokoltsov. "Domains of Quasi Attraction: Why Stable Processes Are Observed in Reality?", *Fractal and Fractional* 7 (2023), 752

[3] Vassili N. Kolokoltsov. "Fractional Equations for the Scaling Limits of Lévy Walks With Position-Dependent Jump Distributions", *Mathematics* 11 (2023), 2566

General kernels and parametrised families: in search of semigroup properties

30 Aug.
15:00
CEST

Arran Fernandez

Eastern Mediterranean University, Northern Cyprus

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 30 August 2024
YouTube: https://youtu.be/Z3jKMA4IkwI

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: operators with general kernels, semigroup property, fundamental theorems of fractional calculus, operational calculus

Abstract: The field of fractional calculus has burgeoned recently with many newly defined operators roughly fitting the same mould: time-fractional operators in one dimension defined by convolution with some kernel function. In order to put some structure on the field, several researchers have suggested categorising these operators into classes defined by some general type of kernel function. However, so many different general kernel functions have been proposed that they too need to be categorised and understood in relation to each other.

Each type of general kernel has its own advantages and disadvantages; I will provide an overview of several proposals from the last fifteen years. Then I will pose a research question – what is the most general kernel function that depends on a "fractional order" parameter α and gives fractional integral operators having a semigroup property in α? – and answer it by using algebraic theory inspired by Mikusiński's operational calculus. The general kernels proposed herein give rise to so-called one-parameter families and two-parameter families of fractional operators. These operators have a natural structure, in which fractional integrals and derivatives can be defined satisfying a fundamental theorem of fractional calculus, and differential equations can be posed and solved within the theory of such operators using operational calculus. Some potential extensions and open problems will also be mentioned.

Biography: Arran Fernandez is an associate professor at the Eastern Mediterranean University in Northern Cyprus. He was educated at the University of Cambridge, where he started as the youngest student in 2010 and graduated as the top student (senior wrangler) in 2013. Following his PhD, also at Cambridge in the Department of Applied Mathematics and Theoretical Physics, he has been working in Northern Cyprus. His research is on the mathematical analysis of fractional calculus, emphasising its connections with other fields of pure mathematics such as Clifford analysis, abstract algebra, analytic number theory, etc. He has published over 70 research papers and serves as an associate editor in 3 mathematical journals.

Bibliography

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A historical survey of fractional calculus with respect to functions and general transmutation relations

23 Aug.
15:00
CEST

Hafiz Muhammad Fahad
School of Natural Sciences, National University of Sciences and Technology, Islamabad, Pakistan

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 23 August 2024
YouTube: https://youtu.be/WVqDPUQ9bps

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: historical survey, fractional calculus with respect to functions, algebraic conjugation, fractional differential equations

Abstract: Derivatives and integrals of one function with respect to another function are well known from basic calculus, using the chain rule and Riemann–Stieltjes integration. The fractional-order versions of these ideas give rise to a theory of fractional calculus with respect to functions, which is often referred to nowadays as ψ-fractional calculus. The history of these operators is longer than most researchers realise, as they have been discovered and re-discovered several times through the decades. In this talk, we trace the full history of fractional calculus with respect to functions. Moreover, we also consider a very general class of fractional calculus operators, given by transmuting the classical fractional calculus along an arbitrary invertible linear operator S. Specific cases of S, such as shift, reflection, and composition operators, give rise to well-known settings such as that of fractional calculus with respect to functions, and allow simple connections between left-sided and right-sided fractional calculus with different constants of differintegration. We define general transmuted versions of the Laplace transform and convolution of functions, and discuss how these ideas can be used to solve fractional differential equations in more general settings.

This is a joint work with Arran Fernandez [1, 2, 3].

Biography: Dr. Hafiz Muhammad Fahad is an Assistant Professor in the Department of Mathematics at the National University of Sciences and Technology, Islamabad, Pakistan. He completed his PhD in 2023 from the Eastern Mediterranean University, Northern Cyprus.

Dr. Fahad's research focuses on special functions, operational and fractional calculus. He investigates the mathematical structure of fractional calculus, connections between different operators and their generalisations, and fractional calculus with respect to functions and corresponding differential equations.

His expertise has earned him invitations to prestigious research programmes organised by institutions like the Isaac Newton Institute for Mathematical Sciences in Cambridge, United Kingdom, and the University of Ghent Analysis & PDE centre in Ghent, Belgium.

Bibliography

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Fractional Calculus for Distributions†

6 Sep.
15:00
CEST

Rudolf Hilfer
ICP, Universität Stuttgart, Allmandrng 3, 70569 Stuttgart, Germany

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 06 September 2024
YouTube: https://youtu.be/S8RUe2CK-Q4

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: fractional calculus, distributions, fractional operator extensions, convolution, Radon measures, maximal domains, precise co-domains, weighted spaces, periodic distributions, discretized fractional calculus, unification of fractional calculus, distributional D'-convolution, Laplace multipliers, operational calculus

† joint work with Tillmann Kleiner

Abstract: Fractional derivatives and integrals are generalized from functions [2, 7] to measures [3] and to distributions [4, 5, 6]. The focus is on translation invariant fractional operators, their domains of definition and their co-domains. It is found that translation-invariant fractional operators interpreted as suitably generalized distributional convolution operators have the largest domains of definition. The extension from domains of functions to distributions also leads to unifications of many previously existing definitions of fractional integrals and derivatives, including discretized fractional calculus operators and fractional operators for periodic distributions [10].

Biography: Rudolf Hilfer has worked on Fractional Calculus and its Applications in Physics since 1985 [1]. In 1990 he introduced fractional derivatives into thermodynamics and into the theory of critical phenomena by identifying the order of the derivative with the order of phase transitions [8]. In 1992 he established time fractional diffusion as the continuum limit of certain continuous time random walks (stochastic processes). This discovery was published during his stay at SISSA in 1994 together with L. Anton [9].

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Inverse source problems for fractional diffusion equations

13 Sep.
15:00
CEST

K. Van Bockstal

Ghent Analysis & PDE Center, Department of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Belgium

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 13 September 2024
YouTube: https://youtu.be/QEzLVtZxnhI

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: inverse source problems, Rothe's method, energy estimates
For keywords are indexing over the last page

Abstract: This talk is based on the papers [1, 2, 3, 4] focused on inverse source problems (ISPs) for time-fractional diffusion equations with time-dependent coefficients in the governing second-order linear elliptic operator. Specifically, I will examine the uniqueness of a solution in determining the space-dependent part of the source from the final-in-time measurement and time-averaged measurement, as presented in [1, 2]. I will also present counterexamples that demonstrate the failure of the uniqueness of a solution when the conditions for uniqueness are not met. Next, I will discuss the reconstruction of the time-dependent part of the source from the knowledge of the space-averaged measurement, as considered in [3]. The main research questions addressed include: (i) the existence and uniqueness of a (weak) solution to the ISP for exact data, and (ii) the numerical reconstruction of the unknown source. Finally, I will discuss the extension of the results to multi-term time-fractional diffusion equation, as presented in [4], and conclude with some potential directions for future work.

Biography: Dr K. Van Bockstal obtained his PhD (in mathematical engineering) in 2015 at Ghent University, Belgium, and is currently a postdoctoral researcher (Ghent Analysis & PDE Center) at the Department of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University. His research interests are related to the mathematical analysis of evolutionary partial differential equations as well as to the development of numerical algorithms and their numerical implementation. This research focus concerns direct and inverse problems in heat transfer, elasticity, electromagnetism and thermo-elasticity. He was awarded the EAIP Young Scientist Award of the 8th International Conference "Inverse Problems: Modelling and Simulation", May 2016.

Bibliography

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Inverse Problems of Determining the Order of Fractional Derivatives in Partial Derivative Equations

20 Sep.
15:00
CEST

Ravshan Ashurov

Institute of Mathematics, Academy of Science of Uzbekistan, Tashkent, Uzbekistan

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 20 September 2024
YouTube: https://youtu.be/FgrAdUFSG1o

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: inverse problems, partial differential equations, initial-boundary value problems

Abstract: Determining the unknown order of a fractional derivative in differential equations modeling various processes is an important problem in modern applied mathematics. Inverse problems of determining these unknown parameters are not only of theoretical interest, but are also necessary for finding a solution to an initial-boundary value problem and studying the properties of solutions. In the last decade, this problem has been actively studied by many specialists. A number of interesting results have been obtained that have a certain applied significance. This report will give a brief overview of the most interesting works in this area, and will also consider methods for solving such inverse problems for equations of mathematical physics.

Biography: Professor Ravshan Ashurov is currently head of laboratory at the Institute of Mathematics of the Academy of Science of Uzbekistan. He studied at Moscow State University, receiving his PhD from there in 1982 and a Doctor of Science also from there in 1992. He has worked as a scientific researcher or visiting scientist at a number of institutions including Birmingham University in England, Vanderbilt University in the US, and the ICTP in Trieste. He has published more than 100 scientific papers as well as several books and monographs in English, Russian, and Uzbek. His research interests include fractional differential equations of ordinary and partial type, spectral theory of differential and pseudo-differential operators, harmonic analysis, and wavelet transforms.

Bibliography

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A method to handle identification problems with unknown history and its application to determination of parameters of generalized fractional diffusion equations

13 Sep.
15:00
CEST

Jaan Janno
Tallinn University of Technology

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 13 September 2024
YouTube: https://youtu.be/PXbYA95Uz04

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: fractional diffusion equation, inverse problems, general fractional derivatives

Abstract: We introduce a method to handle identification problems with unknown history. The method presumes the analyticity after t_0 of an output of a system corresponding to unknown input that was active before t_0. An additional input is generated by an observer after $t_1 > t_0$. The analyticity enables to remove the unknown history and consider identification of parameters of a system independently. We apply the method to three identification problems posed to generalized fractional diffusion equations.

Biography: Jaan Janno is an Estonian professor: born in 1961, graduated from Tartu State University in 1984, obtained the degree of Candidate of Mathematics and Physics (PhD equivalent) at the Institute of Mathematics and Physics of the Ural Branch of the Academy of Sciences of the USSR in 1988, Professor at the Tallinn University of Technology since 2003. He has served on the editorial board of a number of journals and as a plenary speaker in international conferences, and he has been part of multiple research projects funded by the Estonian government.

Bibliography

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04 Oct.
15:00
CEST

Uniqueness and stability results for inverse problems for time-fractional diffusion-wave equations

Masahiro Yamamoto
The University of Tokyo, Japan / Zonguldak Bülent Ecevit University, Turkey

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 04 October 2024
YouTube: https://youtu.be/R8B-NYBuBt0

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: inverse problems, time-fractional differential equations, uniqueness and stability

Abstract: One should be concerned with many kinds of the diffusion of substances in heterogeneous media, and when one considers diffusion of contaminants such as Caesium-137, such studies may be very serious issues for public safety. In the diffusion in heterogeneous media, it is often observed that profiles of density of substances indicate large deviation from the classical diffusion profiles. Therefore, more suitable model equations are demanded and time-fractional diffusion-wave equations is one hopeful model equation and call great attention.

Anomalous diffusion is characterized by slow diffusion in some sense and long-tail profiles, and for fractional equations we can prove slow decay for large time, less smoothing property of distribution, etc. Thus the time-fractional diffusion-wave equation is very probable model:

$$d_t^\alpha u(x,t) - Au(x,t) = F(x,t), \quad x \in \Omega,\ 0 < t < T \tag{1}$$

with initial and boundary conditions. Here we describe most basic form of the equation and will study more general forms. Here Ω is a bounded domain in $\mathbb{R}^d$, $d \in \mathbb{N}$, $-A$ is a uniform elliptic operator of the second order. Moreover d_t^α with $\alpha \in (0,2) \setminus \{1\}$, is a Caputo type of fractional derivative: for $0 < \alpha < 1$, it is defined by

$$d_t^\alpha w(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{dw}{ds}(s)ds$$

if $w \in W^{1,1}(0,T)$, where Γ is the gamma function.

Introducing time-fractional diffusion equations for modelling actual anomalous diffusion, we should first identify parameters governing diffusion. As such parameters, we refer for example to the orders of time-derivative, source terms. This is nothing but inverse problems.

The main purpose of this talk is to study several kinds of inverse problems for time-fractional diffusion-wave equations (1) and present results on the uniqueness and the stability. We mainly study the following kinds of inverse problems:

(I) Determination of orders α and related quantities.

(II) Determination of $\mu(t)$ or $f(x)$ in the case where a source $F(x,t)$ is modelled in a form $F(x,t) = \mu(t)f(x)$.

(III) Backward problems in time: for fixed $T > 0$ determine

$$\begin{cases} u(\cdot,0) \quad \text{by } u(\cdot,T) \text{ if } 0 < \alpha < 1, \\ \{u(\cdot,0),\, \partial_t u(\cdot,0)\} \quad \text{by } \{u(\cdot,T),\, \partial_t u(\cdot,T)\} \text{ if } 1 < \alpha < 2. \end{cases}$$

It is important: which kind of data should we adopt for the uniqueness for inverse problems? Adequate choices of data essentially depend on properties of solutions to initial boundary value problems for time-fractional diffusion-wave equations. Thus our studies for inverse problems should be grounded on the direct problems, and we will explain also necessary knowledge of direct problems. [**?**]

Biography: Professor Masahiro Yamamoto received his Ph.D. from the University of Tokyo in March 1988. He has been a professor at the Graduate School of Mathematical Sciences of the University of Tokyo since April 2010, where he was previously a research associate from April 1985 to March 1990 and an associate professor from April 1990 to March 2010. Since 2024, he has also been a professor at Zonguldak Bülent Ecevit University in Turkey. His research topics include inverse problems for partial differential equations, fractional partial differential equations, and industrial mathematics. He has published almost 400 peer-reviewed papers, and received almost 8,000 citations from more than 3,000 publications (according to the Mathematical Reviews database of the American Mathematical Society).

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11 Oct.
15:00
CEST

Coefficient identification in a space-fractional equation with Abel type operators

Barbara Kaltenbacher
University of Klagenfurt, Austria

Time: 15:00 - 16:00 CEST (Rome / Paris)
Date: 11 October 2024
YouTube: https://youtu.be/wvmIB_3mW3o

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: inverse problems, fractional PDE, coefficient identification, Newton's method

Abstract: We consider the inverse problem of recovering an unknown, spatially-dependent potential $q(x)$ from a fractional order equation $Lu + qu = f$ defined in a two dimensional region from boundary information. Here the differential operator L is a directionally fractional one, based on the Abel fractional integral. In the classical integer derivative case this reduces to recovering q in $-\Delta u + q(x)u = f$, which is a well-studied problem. We develop both uniqueness and reconstruction results and show how the ill-conditioning of this inverse problem depends on the geometry of the region and the fractional orders used in x and y directions [1].

Biography: The main emphasis of my research lies in Inverse Problems and their regularization, where I have co-authored three monographs and contributed more than a hundred journal papers. As this also involves modeling, analysis and numerics of the corresponding forward problems, I got involved in several application related topics such as piezoelectricty, hysteresis in magnetics and ferroelectrics, as well as nonlinear acoustics, where the latter has recently become another focus of my research with a monograph co-authorship and more than thirty journal papers so far.

My main achievements in regularization theory are on iterative regularization of nonlinear problems in Hilbert and also in general Banach spaces, as well as their efficient and stable implementation using and further developing modern techniques from numerics of PDEs and optimization. Concerning nonlinear acoustics, I have contributed to the analysis of classical and also advanced models.

This research has been supported by the Austrian and the German Science Foundations, as well as by industrial partners in a number of projects, that among others provided funding for the fifteen PhD students whom I had the pleasure to supervise so far.

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Multiscale Modelling and Simulation for Anomalous and Nonergodic Dynamics: From Statistics to Mathematics

18 Oct.
16:00-17:00
CEST

Weihua Deng

School of Mathematics and Statistics and State Key Laboratory of Natural Product Chemistry, Lanzhou University

Time: 16:00 - 17:00 CEST (Rome / Paris)
Date: 18 October 2024
YouTube: https://youtu.be/FHtzXnE4W9k

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: multi-scale modelling, anomalous dynamics, deep learning

Abstract: In recent decades, anomalous and nonergodic dynamics are topical issues in almost all disciplines. In 2004, the phrase "anomalous is normal" was used in a title of a PRL paper, which reveals that the diffusion of classical particles on a solid surface has rich anomalous behavior controlled by the friction coefficient, meaning that anomalous dynamics phenomena are ubiquitous in the natural world. This talk first introduces the dynamics from a physical and atomistic way, by considering the random walk of the diffusing particles, then derives the partial differential equations with integral-differential operators governing the PDFs of the various statistical observables. Finally, we discuss the (traditional and deep learning based) numerical methods for the newly build PDEs.

Biography: Dr Deng is a professor of School of Mathematics and Statistics and State Key Laboratory of Natural Product Chemistry, Lanzhou University. His reseaech interests include multiscale modelling, scientific computation, deep learning, and their applications in chemistry and biology.

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Learning Nonlocal Operators

25 Oct.
16:00-17:00
CEST

Yue Yu
Lehigh University

Time: 16:00 - 17:00 CEST (Rome / Paris)
Date: 25 October 2024
YouTube: https://youtu.be/l3l_kmDI1-8

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: scientific machine learning, peridynamics, nonlocal kernel regression

Abstract: Nonlocal models, including peridynamics and fractional PDEs, often use integral operators that embed length-scales in their definition. However, the integrands especially the kernels in these operators are difficult to define from the data that are typically available for a given physical system, such as laboratory mechanical property tests.

In this talk, we will consider learning of complex material responses as an exemplar problem to investigate automated nonlocal model discovery from experimental data. In particular, we propose to parameterize the mapping between excitation and corresponding system responses in the form of nonlocal operators, and infer the integral kernels from experimental measurements. As such, the model is built as mappings between infinite-dimensional function spaces, and the learnt models are resolution-agnostic. Moreover, the nonlocal operator architecture also allows the incorporation of fundamental mathematical and physics knowledge. Both properties improve the learning efficacy and robustness from scarce measurements.

To demonstrate the applicability of our nonlocal operator learning framework, two typical scenarios will be discussed: (1) learning of a material-specific constitutive law, and (2) development of a foundation constitutive law across multiple materials.

Biography: Yue Yu received her B.S. from Peking University in 2008, and her Ph.D. from Brown University in 2014. She was a postdoc fellow at Harvard University after graduation, and then she joined Lehigh University as an assistant professor of applied mathematics and was promoted to full professor in 2023. Her research lies in the area of numerical analysis and scientific computing, with recent projects focusing on nonlocal problems and scientific machine learning. She has received an NSF Early Career award and an AFOSR Young Investigator Program (YIP) award.

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Super slow diffusion: analysis and computation

01 Nov.
13:00-14:00
CET

Changpin Li
Shanghai University, Shanghai 200444, China

Time: 13:00 - 14:00 CET (Rome / Paris)
Date: 01 November 2024
YouTube: https://youtu.be/Cm-GXPtFX_s

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: super slow diffusion, Hadamard derivative, asymptotics, numerical computation

Abstract: The super slow diffusion indicates that the solution to the diffusion equation has algebraic asymptotics in the sense of logarithmic function. It is modelled by Hadamard derivatives. In this talk, we mainly introduce the logarithmic asymptotics and regularity of the solution to the Caputo-Hadamard fractional evolution equation. Based on these theoretical analysis, we construct the reliable numerical algorithms to numerically solve it, where various kinds of numerical algorithms for Caputo-Hadamard derivatives are also displayed.

Biography: Changpin Li earned his doctoral degree in computational mathematics from Shanghai University in 1998. He worked in the same univeristy and became full professor in 2007. He has been Chien Wei-zang Scholar (II) since 2020 and FIMA (Fellow of the Institute of Mathematics and its Applications, UK) since 2021. His main research interests include numerical methods for fractional partial differential equations and dynamics of fractional differential equations. He has presided over 8 NSFCs (National Natural Science Foundations of China). And he has published three monographs and more than 150 papers in SIAM jounrals, IMA J Appl Math, J Comput Phys, J Nonlinear Sci, J Sci Comput, PRE, Phys D, etc.

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Numerical methods for the spectral fractional Helmholtz equation

08 Nov.
15:00-16:00
CET

Lehel Banjai
Maxwell Institute for Mathematical Sciences,
School of Mathematical & Computer Sciences, Heriot-Watt University, UK

Time: 15:00 - 16:00 CET (Rome / Paris)
Date: 08 November 2024
YouTube: https://youtu.be/vZKxUTVz-sQ

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: spectral fractional Laplacian, fractional Helmholtz, hp-FEM, contour integral methods, sinc quadrature

Abstract: In this talk we consider the Helmholtz equation where the usual Laplacian is replaced by a non-local variant: the spectral fractional Laplacian. The thus obtained fractional Helmholtz equation has applications to geophysical electromagnetics [1], but in this talk we focus on the development and analysis of efficient numerical methods. We describe and analyse two possible approaches. Firstly, a direct contour integral method discretised by a sinc quadrature combined with an hp-finite element method [2]. We will discuss the merits of choosing the contour before discretisation of the Laplacian or after. Secondly a tensor product hp-finite element method applied to an equivalent local problem in a spatially extended domain [3,4]. In both cases we show that the cost is equivalent to solving a single standard Helmholtz equation and a fractional Poisson problem. Relationship to existing works is described [1,5]. The talk ends with numerical experiments showing exponential convergence of both approaches.

Biography: Lehel Banjai is a professor at the Mathematics Department of Heriot-Watt University which is part of the Maxwell Institute for Mathematical Sciences in Edinburgh, UK. Before coming to Edinburgh, he was at the Max Planck Institute for Mathematics in the Sciences, Leipzig and at the University of Zurich. He obtained his D.Phil. in the Numerical Analysis Group at the University of Oxford.

He is best known for his work on numerical methods for time-domain boundary integral equations and on convolution quadrature (see the recent research monograph with Francisco Javier Sayas). In recent years he has been active in researching numerical methods for fractional differential equations – time and space, a priori and a posteriori analysis. His other interests include numerical wave propagation, data sparse methods such as H-matrices and FMM, conformal mapping etc.

He has jointly founded and co-organised the One World Numerical Analysis Seminar and is currently acting as an Associate Editor for SINUM.

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Peridynamic differential operator and its applications

15 Nov.
15:00-16:00
CET

Erdogan Madenci
University of Arizona Tucson,
Arizona, USA

Time: 15:00 - 16:00 CET (Rome / Paris)
Date: 15 November 2024
YouTube: https://youtu.be/u0srP6JO8bU

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: nonlocal, peridynamics, partial differential equations, discontinuity

Abstract: Peridynamic (PD) differential operator (PDDO) converts the existing governing field equations from their local to nonlocal form while introducing an internal length parameter. The PDDO enables differentiation through integration. As a result, the equations become valid everywhere regardless of discontinuities. The lack of an internal length parameter in the classical form of the governing equations is the source of problem when addressing discontinuities. Although the PD theory is extremely suitable to model the response of structures involving crack initiation and propagation at multiple sites, with arbitrary paths, it is also applicable to other field equations involving phase change arising from corrosion and electrodeposition. Furthermore, it is applicable to hyperbolic equations for which the solution does not smooth out with time and discontinuities persist such as a shock wave. Lastly, the PDDO enables the evaluation of derivatives of any order in a multi-dimensional space and provides a unified approach to transfer information within a set of discrete data, and among data sets. This presentation provides an overview of the PD concept, the derivation of the PDDO, and applications concerning data reduction and recovery, solutions to complex partial differential equations, failure prediction in structural materials under complex loading conditions.

Biography: Erdogan Madenci has been a professor in the Department of Aerospace and Mechanical Engineering at the University of Arizona since 1989. Prior to joining the University of Arizona, he worked at Northrop Corporation, Aerospace Corporation, and the Fraunhofer Institute. Also, he worked at the KTH Royal Institute of Technology, NASA Langley Research Center, Sandia National Labs and MIT as part of his sabbatical leaves. He is the lead author of six books on Peridynamics (available in Chinese and Persian) and Finite Element analysis. He serves as the Co-Editor-in-Chief of the Journal of Peridynamics and Nonlocal Modeling and an Associate Editor of ASME Open Journal of Engineering. He is a Fellow of ASME and an Associate Fellow of AIAA.

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On a new class of variable-order fractional operators: theory, applications and numerical methods

15 Nov.
16:00-17:00
CET

Roberto Garrappa
University of Bari, Italy

Time: 16:00 - 17:00 CET (Rome / Paris)
Date: 15 November 2024
YouTube: https://youtu.be/yGh6Ndb5i2Q

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: fractional derivatives, variable order, Laplace transform

Abstract: A new and different approach to generalize fractional-order operators from constant to variable order has been recently proposed and investigated [1]. The new variable-order operators find inspirations on some pioneering works by the Italian engineer Giambattista Scarpi and dating back to the early seventies [2]. The main difference with other variable-order operators is that the generalization from constant to variable order is made in the Laplace transform domain rather than directly in the time domain, with an important advantage: based on the Sonine condition, once the integral operator has been defined, the construction of the corresponding differential operator, ensuring inversion of the integral, is straightforward.

The numerical treatment of fractional differential equations with this family of variable-order operators is not trivial and requires methods constructed to operate in the Laplace transform domain. In this talk, we first review the main mathematical aspect of the proposed operators, we briefly discuss some applications [3,4], and hence we treat with more details some numerical aspects related to solving variable-order fractional differential equations [5].

Biography: Roberto Garrappa is a full professor of Numerical Analysis at the University of Bari. His main research interest focuses on the numerical solution of differential equations of fractional order and on the analysis and numerical evaluation of special functions. He is author of more than 70 scientific papers and of a series of Matlab codes for fractional-order problems freely available on the Mathworks website.

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Dealing with the Singularity from Fractional Laplacian with Spectral Methods

22 Nov.
15:00-16:00
CET

Zhongqiang Zhang

Department of Mathematical Sciences, Worcester Polytechnic Institute

Time: 15:00 - 16:00 CET (Rome / Paris)
Date: 22 November 2024
YouTube: https://youtu.be/TClWpXH_pZg

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: integral fractional Laplacian, regularity, singularity structure, low-order terms

Abstract: We study certain fractional elliptic equations that include lower-order terms. It is well-known that the fractional Laplacian introduces a mild singularity near the boundary of the bounded domain. Despite this, adding lower-order terms does not reduce the solution's smoothness in Hölder and Sobolev spaces, either near the boundary or within the domain. However, numerical methods can suffer in accuracy. This is particularly the case when the weak boundary singularity is explicitly incorporated into the computed solution. We demonstrate that this occurs with spectral methods that use weighted Jacobi polynomials for computations on an interval. We also find similar effects for spectral Galerkin methods on a disk. Additionally, we conjecture that the pattern of singularities in these solutions is similar to the structure in Mittag-Leffler functions.

Biography: Zhongqiang Zhang is an Associate Professor of Mathematics at Worcester Polytechnic Institute. His research interests include numerical methods for stochastic and integral differential equations, computational probability, and mathematics for machine learning. Before he joined in Worcester Polytechnic Institute in 2014, he received Ph.D. degrees in mathematics at Shanghai University in 2011 and in applied mathematics at Brown University in 2014. He co-authored a book with George Karniadakis on numerical methods for stochastic partial differential equations with white noise.

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Scalable methods for nonlocal models

29 Nov.
15:00-16:00
CET

Christian Glusa
Sandia National Laboratories

Time: 15:00 - 16:00 CET (Rome / Paris)
Date: 29 November 2024
YouTube: https://youtu.be/zJ5kwMJg2s0

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: fractional Laplacian, multigrid, hierarchical matrices

Abstract: The naive discretization of nonlocal operators leads to matrices with significant density, as compared to classical PDEs. This makes the efficient solution of nonlocal models a challenging task. In this presentation, we will discuss ongoing research into efficient hierarchical matrix assembly and geometric and algebraic multigrid preconditioners that are suitable for nonlocal models.

Biography: Christian Glusa is a staff member in the Center for Computing Research at Sandia National Laboratories. His interests include scalable linear solvers such as multigrid and domain decomposition methods, and their application to problems in electromagnetics and fractional order equations.

Christian received a PhD in applied mathematics from Brown University in 2017, and undergraduate degrees from Karlsruhe Institute of Technology (KIT) and Ecole Polytechnique in 2012

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Analysis and computation for variable-exponent fractional problems

06 Dec.
15:00-16:00
CET

Hong Wang
University of South Carolina

Time: 15:00 - 16:00 CET (Rome / Paris)
Date: 06 December 2024
YouTube: https://youtu.be/hFKqEB4QrR0

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**
* SISSA, International School of Advanced Studies, Trieste, Italy
** Eastern Mediterranean University, Northern Cyprus

Keywords: fractional calculus, variable exponent, mathematical analysis, numerical method

Abstract: Fractional differential equations of variable exponent provide adequate descriptions for complex phenomena, while rigorous mathematical and numerical analysis are far from well developed. We discuss several methods to treat typical variable-exponent fractional problems such as the time-fractional mobile-immobile diffusion equation, subdiffusion equation and fractional diffusion-wave equation. The well-posedness and regularity of these models are proved, and numerical analysis is accordingly performed. Furthermore, optimal control of variable-exponent fractional problems is proposed and analyzed.

Biography: Hong Wang is a professor at University of South Carolina. His research interest includes numerical analysis and scientific computing in porous medium flow and fractional problems. He has more than 200 publications on journals such as SIAM J. Sci. Comput., SIAM J. Numer. Anal., SIAM J. Control Optim. and Multiscale Model. Simul..

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Coupling Peridynamics and Classical Continuum Mechanics: Overview, Methods, and Analysis

06 Dec.
16:00-17:00
CET

Pablo Seleson

Oak Ridge National Laboratory

Time: 16:00 - 17:00 CET (Rome / Paris)
Date: 06 December 2024
YouTube: https://youtu.be/rrpsn9-neJE

Hosted at: SISSA, International School of Advanced Studies, Trieste, Italy

Organizers: Pavan Pranjivan Mehta* and Arran Fernandez**

* SISSA, International School of Advanced Studies, Trieste, Italy

** Eastern Mediterranean University, Northern Cyprus

Keywords: peridynamics, local-to-nonlocal coupling, fracture

Abstract: Peridynamics is a powerful nonlocal reformulation of classical continuum mechanics, suitable for material failure and damage simulation. Peridynamic models do not require spatial differentiability assumptions of displacement fields, as opposed to classical constitutive relations, leading to natural representation of material discontinuities such as cracks. However, the nonlocality of peridynamics results in a significant increase in computational cost, compared to classical (local) continuum mechanics simulations. Coupling peridynamics and classical continuum mechanics is a popular technique to combine the strengths of nonlocal and local representations and attain efficient and accurate solutions of peridynamics problems. A main issue in local-to-nonlocal coupling is the appearance of coupling artifacts around coupling interfaces, which can pollute the solutions of coupled problems, and different methods were proposed to overcome this challenge. This talk will provide an overview of coupling peridynamics and classical continuum mechanics, discuss some coupling methods, and present a coupling analysis.

Biography: Dr. Pablo Seleson is a Research Scientist in the Computer Science and Mathematics Division at the Oak Ridge National Laboratory, where he began as an Alston S. Householder Fellow. Dr. Seleson received both his Bachelor's degree in Physics and Philosophy and his Master's degree in Physics from the Hebrew University of Jerusalem in 2002 and 2006, respectively, and his Ph.D. in Computational Science from Florida State University in 2010 under the advisement of Prof. Max Gunzburger. After graduation, he joined the Institute for Computational Engineering and Sciences (ICES) at The University of Texas at Austin as an ICES Postdoctoral Fellow under the supervision of Prof. J. Tinsley Oden, where he also worked in close collaboration with the Computer Science Research Institute at Sandia National Laboratories. Dr. Seleson's research focuses on multiscale materials modeling and on mathematical and computational analysis of peridynamics and related nonlocal models with application to computational fracture modeling. Dr. Seleson is an Associate Editor of Applicable Analysis and editorial board member of the Journal of Peridynamics and Nonlocal Modeling, a Member-at-Large of the Executive Committee of the U.S. Association for Computational Mechanics (USACM), past Chair of the USACM Technical Thrust Area (TTA) on Large Scale Structural Systems and Optimal Design, Vice-Chair of the USACM TTA on Mathematical

Methods in Computational Engineering & Sciences, and Member of the Computational Mechanics Committee of the Engineering Mechanics Institute (EMI) of the American Society of Civil Engineers (ASCE).

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Author Index

Keyword Index

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