

## MINI WORKSHOP

# ADVANCES IN NUMERICAL ANALYSIS, SCIENTIFIC COMPUTING AND COMPUTATIONAL MECHANICS

**July 13, 2015: Room A-128 (1<sup>st</sup> Floor)**

**Welcome and Introduction 14:15-14:30**



**Virtual Element Spaces - Franco Brezzi (IUSS, Pavia – IMATI-CNR), 14:30-15:20**

**[Mathematical Analysis, Modelling and Applications Seminar]**

We present some general families of  $H(\text{div})$ -conforming and  $H(\text{curl})$ -conforming Virtual Element Spaces on polygonal and polyhedral decompositions. These spaces could be used, together with the more classical  $H^1$  conforming Virtual Elements (see [1], [2]) and with the usual discontinuous piecewise polynomial spaces, in order to approximate boundary value problems for Partial Differential Equations in mixed formulation. These spaces generalize the Mixed Virtual Elements of [3], and previous Mimetic Finite Differences for Mixed Formulations. (see e.g. [4]).

### REFERENCES

- [1] Beirão da Veiga L., Brezzi F., Cangiani A., Manzini G., Marini L.D., Russo A. (2013) Basic Principles of Virtual Element Methods. Math. Models Methods Appl. Sci. 23(1): 199-214.
- [2] Ahmed B., Alsaedi A., Brezzi F., Marini L.D., Russo A. (2013) Equivalent Projectors for Virtual Element Methods. Comput. Math. Appl. 66 (3): 376-391.
- [3] Brezzi F., Falk R.S., Marini L.D (2014) Basic Principles of Mixed Virtual Element Methods. ESAIM Mathematical Models and Numerical Analysis. 48 (4): 1227-1240.
- [4] Beirão da Veiga L., Lipnikov K, Manzini G., The Mimetic Finite Difference Method for Elliptic Problems, Springer, MS&A, Vol 11.

**Virtual Element Methods for Advection-Diffusion Problems – Donatella Marini (University of Pavia, IMATI-CNR), 15:20-16:10 [SISSA mathLab seminar]**

We apply the Virtual Element approach (see [1],[2]) for the discretization of advection-diffusion-reaction problems with variable coefficients on polygonal grids. We analyze both the primal and the mixed formulation, and prove optimal convergence results in  $H^1$  and in  $L^2$ . We provide numerical tests confirming the theoretical results.

### REFERENCES

- [1] Beirão da Veiga L., Brezzi F., Cangiani A., Manzini G., Marini L.D., Russo A. (2013) Basic Principles of Virtual Element Methods. Math. Models Methods Appl. Sci. 23(1): 199-214.
- [2] Ahmed B., Alsaedi A., Brezzi F., Marini L.D., Russo A. (2013) Equivalent Projectors for Virtual Element Methods. Comput. Math. Appl. 66 (3): 376-391.

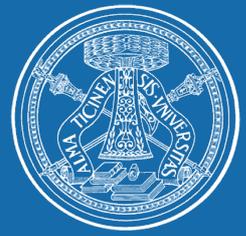
**Break 16:10-16:40**

**Fluid-structure interaction problems: a new variational formulation for FE-IBM – Lucia Gastaldi (University of Brescia), 16:40-17:30 [GNCS keynote seminar]**

We present some recent results on the finite element approximation of fluid-structure interaction problems. In particular, we consider a new variational formulation of the Immersed Boundary Method (IBM) based on the introduction of a suitable Lagrange multiplier. We prove that a semi-implicit time advancing scheme is unconditionally stable. At each time step we have to solve a saddle point problem. We study existence and uniqueness of the continuous solution and analyze the finite element discretization providing optimal error estimates.

**Distributed Lagrangian Multiplier Finite Elements Immersed Boundary Method – Nicola Cavallini (SISSA mathLab - MHPC) 17:30-18:00 [GNCS focus seminar]**

Immersed Boundary Method has been introduced by C. S. Peskin in the seventies relying on a finite difference discretisation. The method is focused on fluid structure interactions applied to biomechanics problems. In the early 2000's D. Boffi e L. Gastaldi reformulated the problem in a finite elements framework. In this formulation the elastic stress contribution to the motion is treated explicitly. Recently D. Boffi, L. Gastaldi ed N. Cavallini introduced a Distributed Lagrangian Multiplier that implicitly couples fluid and structure. In this talk we are going to present the latest applications on this topic. The goal is to highlight the key features of this formulation. Particular care will be devoted to stability, robustness, and conservation properties of the method.



**July 14, 2015: Room A-004 (Ground Floor)**

**9:30 Opening remarks**

**Adaptive finite element approximation of mixed eigenvalue problems – Daniele Boffi (University of Pavia), 9:40-10:30 [GNCS keynote seminar]**

We show that the h-adaptive mixed finite element method for the discretization of the eigenvalues of Laplace operator produces optimal convergence rates in terms of nonlinear approximation classes. The results are valid for the typical mixed spaces of Raviart-Thomas or Brezzi-Douglas-Marini type with arbitrary fixed polynomial degree in two and three dimensions. Our theory is cluster robust, in the sense that it allows for the simultaneous optimal approximation of the eigenvalues belonging to the same cluster. This is a joint work with D. Gallisti, F. Gardini, and L. Gastaldi.

**Large-Scale Aerodynamic Shape Optimization using POD-based reduced-order modeling – Angela Scardigli (Politecnico di Torino) 10:30-11:00 [GNCS focus seminar]**

The definition of a reduced-order model (ROM) to be used as surrogate model during the aerodynamic optimization of a complex constrained geometry is addressed. In order to avoid handling geometry variation within the ROM, we employ a hybrid low-order/high order method which combines a POD-based reduced-order model and a canonical CFD solver using a domain decomposition approach. Generally speaking this method delegates non-linear effects to the canonical solver, used within a crucial region, whereas linear phenomenology is addressed by the ROM. The two models are then coupled through a Schwarz method exploiting directly the properties of the POD basis. A method for determining the interface between the two subdomains is therefore discussed. The effectiveness and drawbacks of this approach are highlighted on a large-scale aerodynamic problem, i.e. the mainsail thrust optimization of a sailing boat.

**Break 11:00-11:30**

**Convergence and optimality of adaptive hp-fem – Claudio Canuto (Politecnico di Torino), 11:30-12:20 [GNCS keynote seminar]**

We consider the hp-version of the finite element method for the solution of linear elliptic problems. We design an adaptive scheme which hinges on a recent algorithm by P. Binev for adaptive hp-approximation. We prove convergence with contraction rate, and we investigate the optimality properties of the algorithm. This is a joint work with R.H. Nochetto (Maryland), R. Stevenson (Amsterdam), and M. Verani (Politecnico di Milano).

**IGA-based reduced order model tool for viscous flows in parametrized shapes – Filippo Salmoiraghi (SISSA mathLab), 12:20-12:50 [GNCS focus seminar]**

We provide a new complete concept "tool" to deal with parametrized shapes managed by free-form deformation techniques into an isogeometric setting. This tool is totally integrated into model order reduction techniques, developed for viscous flow in parametrized shapes. This environment has been created in the framework of the project UBE -Underwater Blue Efficiency- for the optimization of immersed parts of motor yachts, including exhausting flows devices. The study is benefitting of several properties of reduced order modeling approaches such as offline-online calculations, parametric design.

**Lunch Break 13:00-14:30 (SISSA Cafeteria/Restaurant/Gardens)**

**Reduced order modelling of bifurcation problems with applications to incompressible fluid mechanics – Giuseppe Pitton (SISSA mathLab – AMMA), 14:30-15:00 [GNCS focus seminar]**

Nonlinear parametrized Partial Differential Equations (PPDEs) arise frequently in the mathematical modelling of physical systems. A common issue in this kind of problems is the possible loss of uniqueness of the solution as the parameters are varied and cross a singular point. In the present work, the numerical detection of singular points is performed through a Reduced Basis Method, coupled with a Spectral Element Method for the numerically intensive computations. We will present example applications on laminar fluid mechanics problems, where pitchfork, hysteresis and Hopf bifurcation points are detected.

**Reduced order models for parametrized problems in computational fluid dynamics – Francesco Ballarin (SISSA mathLab), 15:00-15:30 [GNCS focus seminar]**

In this talk we discuss a computational reduction framework for parametrized problems in computational fluid dynamics, based on a offline/online splitting between an high fidelity model (offline) and a reduced order one (online). Special attention will be devoted to the stabilization of the resulting online system, and shape parametrization maps to efficiently handle deformation of the computational domain. In the second part of the talk we will show some applications of the proposed framework to optimal control problems, fluid-structure interaction and a focus on cardiovascular flows.

**Conclusions and Round/Table, Luca Heltai, Gianluigi Rozza 15:30-16:00**